EQUATION OF DYNAMIC PLASTICITY FOR POLYCRYSTALLINE METALLIC MATERIALS

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A theoretical investigation was made of the inelastic behavior of polycrystalline metallic materials on the basis of dislocation representations, developed in [1-3]. A kinetic equation of one-dimensional deformation of a medium was obtained in [1, 2]. In the present article an evaluation is made of a possible variant of the generalization of the one-dimensional equations of motion of a medium with structural defects for the case of three-dimensional deformation.

1. In a wide range of change in the deformation rates and temperatures, the microstructural mechanisms of the dynamics of dislocations determine the inelastic behavior of metallic materials. In the general case, the density of the dislocations is described by a tensor of the second rank [4]. However, for media with a chaotic orientation of monocritical grains, it can be assumed that the density is described by the scalar parameter n. Thus, in addition to the determining parameters of the medium ($\hat{\varepsilon}_e$ is the tensor of the elastic deformations and s is the entropy), used in the thermodynamics of elastic media, internal parameters are introduced, characterizing the changes of the structure during the deformation process. If the formation of defects in the continuity of the material (cracks, micropores) are not considered, then, for polycrystalline metallic aggregates, the change in the structure can be characterized by the mean density of the dislocations (the mean number n of dislocation lines intersecting a unit area) and a parameter characterizing the granularity of the material $\varkappa$. With such a choice of the determining parameters of the medium, as the thermodynamic potential it is convenient to take the function of the internal energy

$$U = U(\hat{\varepsilon}_e, s, n, \varkappa).$$

The first origin of the thermodynamics is written in the form [5]

$$dU = \mathrm{Sp}\frac{\hat{\sigma}}{\rho}\,d\hat{\varepsilon} + dq^e, \tag{1.1}$$

where $\hat{\sigma}$ is the stress tensor; ρ is the instantaneous density of the material; $\hat{\varepsilon}$ is the tensor of the total deformations; q^e is the external heat influx. Further, we shall assume the additivity of the rates of elastic and plastic deformations

$$d\hat{\varepsilon}/dt = d\hat{\varepsilon}_e/dt + d\hat{\varepsilon}_p/dt,$$

as well as the incompressibility of the plastic deformations

$$\mathrm{Sp}\hat{\varepsilon}_p = 0.$$

The second origin of the thermodynamics is written in the form

$$Tds = dq' + dq^e, \tag{1.2}$$

q' is the uncompensated heat. We take

$$dq' = \beta\,\mathrm{Sp}\frac{\hat{\sigma}}{\rho}\,d\hat{\varepsilon}_p, \tag{1.3}$$

where β is the fraction of the work in plastic deformations going over into heat; here $0 < \beta < 1$. Equation (1.1), taking account of (1.2), (1.3), can be rewritten in the form

$$dU = \mathrm{Sp}\frac{\hat{\sigma}}{\rho}\,d\hat{\varepsilon}_e + \mathrm{Sp}\frac{\hat{\sigma}}{\rho}\,d\hat{\varepsilon}_p - \beta\,\mathrm{Sp}\frac{\hat{\sigma}}{\rho}\,d\hat{\varepsilon}_p. \tag{1.4}$$

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We write relationship (1.4) in the form

$$\mathrm{Sp}\frac{\partial U}{\partial\hat{\varepsilon}_e}d\hat{\varepsilon}_e+\frac{\partial U}{\partial s}ds+\frac{\partial U}{\partial n}dn+\frac{\partial U}{\partial\varkappa}d\varkappa=\mathrm{Sp}\frac{\hat{\sigma}}{\rho}d\hat{\varepsilon}_e+Tds+(1-\beta)\,\mathrm{Sp}\frac{\hat{\sigma}}{\rho}d\hat{\varepsilon}_p.$$

From the last equality there follow the equations of state

$$\hat{\sigma}=\partial U/\partial\hat{\varepsilon}_e,\ T=\partial U/\partial s,$$

$$1-\beta=\rho\left(\frac{\partial U}{\partial n}\frac{dn}{dt}+\frac{\partial U}{\partial\varkappa}\frac{d\varkappa}{dt}\right)\Big/\mathrm{Sp}\hat{\sigma}\frac{d\hat{\varepsilon}_p}{dt},$$

where $(1-\beta)$ is the fraction of the work in the plastic deformations expended for formation of defects, their growth, and motion [3].

For determination of the fraction of work in plastic deformations, expended for the formation of the defects of their structure and their dynamics, on the basis of determining physical considerations, we must write the kinetic equations

$$dn/dt=f_n(\hat{\varepsilon}_e,s,n,\varkappa),\ d\varkappa/dt=f_\varkappa(\hat{\varepsilon}_e,s,n,\varkappa)$$

and the equation for the tensor of the rates of plastic deformations

$$d\hat{\varepsilon}_p/dt=\hat{J}(\hat{\varepsilon}_e,s,n,\varkappa). \tag{1.5}$$

A one-dimensional analysis of the deformation, creep, and superplasticity of metallic materials, given in [1, 2], makes it possible to write a kinetic equation for the mean density of the dislocations in the form

$$dn/dt=mbnu_s-a_dbnu_d, \tag{1.6}$$

where $m = m(T, \varkappa)$ is the coefficient of multiplication; b is a Burgers vector for unit translation; $u_s = u_s(\tau, T, n, \varkappa)$ is the mean conservation rate of slip of the dislocations; $a_d = a_d(T, \varkappa)$ is the coefficient of annihilation of dislocations of different signs as a result of diffusion; $u_d = u_d(\tau, T, n, \varkappa)$ is the nonconservative velocity of the motion of the dislocations (the diffusional component of the velocity). The first term in the right-hand part of Eq. (1.6) describes multiplication of the dislocations as a result of multiple transverse slip and determines the hardening of the material, and the second term describes the annihilation of the dislocations of different signs due to diffusion at short distances, and determines the recovery of the mechanical properties of the material (relaxation); $\tau = (\sigma_1 - \sigma_2)/2$ [2]; σ_1 and σ_2 are the components of the stress tensor.

During the process of the deformation of polycrystalline materials under high-temperature conditions, there can be growth of the grains. In accordance with the theory of the growth of grains, developed in [6], we have

$$d\varkappa/dt=\frac{1}{2}G\theta b^2u_d\left(\frac{1}{\varkappa_c}-\frac{1}{\varkappa}\right),$$

where θ is a numerical coefficient ($\theta = 0.5$–1); G is the elastic shear modulus; $\varkappa_c$ is the critical radius of a grain, after which it starts to grow. In a wide range of change in the deformation rates and the temperatures we can set $\varkappa = \mathrm{const}$.

For the case of three-dimensional deformation of polycrystalline materials, we take the kinetic equations in the form

$$dn/dt=m(T,\varkappa)bnu_s(\tau_m,T,n,\varkappa)-a_d(T,\varkappa)bnu_d(\tau_m,T,n,\varkappa); \tag{1.7}$$

$$d\varkappa/dt=\frac{1}{2}G\theta b^2u_d(\tau_m,T,n,\varkappa)\left[\frac{1}{\varkappa_c}-\frac{1}{\varkappa}\right], \tag{1.8}$$

where $\tau_m = \sqrt{{}^1/_2\mathrm{Sp}\hat{\tau}^2}$ is the intensity of the maximal shear stresses; in the case of one-dimensional deformation, $\tau_m = \tau$; $\hat{\tau}$ is the tensor of the maximal shear stresses.

2. In accordance with the Cayley–Hamilton theorem [5], the tensor function (1.5) can be represented in the following manner:

$$\hat{J}=A\hat{I}+B\hat{\varepsilon}_e+C\hat{\varepsilon}_e^2, \tag{2.1}$$

where the coefficients A, B, and C depend on the basic invariants of the tensor $\hat{\varepsilon}_e$ and the parameters s, n, $\varkappa$. As a result of the smallness of the elastic deformations, the third term in relationship (2.1) can be disregarded

$$\hat{J} = A\hat{I} + B\hat{\varepsilon}_e. \qquad (2.2)$$

We shall further assume that, in a small physical volume, there is an identical number of dislocations of opposite sign so that the total field of the stresses created by these dislocations is equal to zero. For small elastic deformations, we limit ourselves to the principal linear part of the equation of state [3]

$$\hat{\varepsilon}_e = \frac{1+\nu}{E}\hat{\sigma} - \frac{\nu}{E}\hat{I}\,\mathrm{Sp}\hat{\sigma} + \alpha(T - T_0), \qquad (2.3)$$

where α is the coefficient of linear thermal expansion. Using relation (2.3), from (2.2) we obtain

$$\hat{J} = A'\hat{I} + B'\hat{\sigma},\ A' = A - \frac{B\nu}{E}\mathrm{Sp}\hat{\sigma} + B\,\alpha(T - T_0),\ B' = \frac{B(1+\nu)}{E}. \qquad (2.4)$$

Taking account of the plastic incompressibility of the material, we obtain a connection between the coefficients A' and B'

$$A' = B'p. \qquad (2.5)$$

Substituting (2.5) into (2.4) we obtain

$$d\hat{\varepsilon}_p/dt = \hat{\sigma}'/\mu,\ \mu = 1/B', \qquad (2.6)$$

where $\hat{\sigma}'$ is the deviator part of the stress tensor; μ is the coefficient of the shear viscosity. Introducing the intensity of the rates of the maximal plastic shears

$$d\gamma_m^p/dt = \sqrt{\tfrac{1}{2}\,\mathrm{Sp}\dot{\hat{\gamma}}_p},$$

where $\dot{\hat{\gamma}}_p$ is the tensor of the rates of the maximal plastic shears, from (2.6) we obtain an expression for μ:

$$\mu = \tau_m/(d\gamma_m^p/dt). \qquad (2.7)$$

Substituting (2.7) into (2.6), we obtain

$$d\hat{\varepsilon}_p/dt = \frac{\hat{\sigma}'}{\tau_m}\left(d\gamma_m^p/dt\right). \qquad (2.8)$$

Relation of type (2.8) in different theories of plasticity have been obtained on the basis of other assumptions [7]. Here we use the assumptions of the smallness of the elastic deformations and the incompressibility of the plastic deformations, and the connection (1.5) is postulated.

In the case of monoaxial deformation of samples of a polycrystalline metallic material, in [1, 2] it was assumed that the slip plane of the dislocations coincide with the plane of the action of the maximal shear stresses, and that the Orovan equation, connecting the rate of the maximal plastic shears with the maximal shear stress and the kinetic parameters, was written in the form [1]

$$d\gamma_m^p/dt = bnu_s(\tau, T, n, \varkappa),\ \gamma_m^p = \frac{1}{2}(\varepsilon_{1p} - \varepsilon_{2p}),$$

where ε_{1p} and ε_{2p} are the components of the tensor of the plastic deformation. For the case of the three-dimensional deformation of polycrystalline materials, we generalize the Orovan equation in the form

$$d\gamma_m^p/dt = bnu_s(\tau_m, T, n, \varkappa).$$

3. We consider a body under the action of external forces in a state of equilibrium occupying a small simply connected volume in Euclidean space x_1, x_2, x_3. Let the stressed state of the body be obtained by the deformation $\mathbf{x} = \mathbf{x}(\mathbf{a})$, det $(\partial\mathbf{x}/\partial\mathbf{a}) > 0$ from some initial state. At the moment of time t, the body is loaded elastically. Under these circumstances, it undergoes the deformation $\xi = \xi(\mathbf{x})$. In terms of infinitely small vectors of the tangential spaces at the points $\mathbf{a}$, $\mathbf{x}$, and $\boldsymbol{\xi}$, we can write

$$d\mathbf{x} = \hat{X}d\mathbf{a},\ d\mathbf{x} = \hat{X}_e d\boldsymbol{\xi},$$

where $X = \partial\mathbf{x}/\partial\mathbf{a}$ and $\hat{X}_e = \partial\mathbf{x}/\partial\boldsymbol{\xi}$ are the transformation matrices. The residual deformation is characterized by the matrix

$$\hat{X}_p = \hat{X}_e \hat{X}_*$$

Then, the matrix of the total transformation is represented in the form of the product of the matrices of the elastic and plastic deformations

$$\hat{X} = \hat{X}_e \hat{X}_p.$$

From the last equality there follows an equation for the evolution of the loaded state during the relaxation process ($d\hat{X}/dt = 0$)

$$d\hat{X}_e/dt = \hat{\Omega}^{\cdot}_p \hat{X}_e, \ \hat{\Omega}^{\cdot}_p = -\hat{X}_e \frac{d\hat{X}_p}{dt} \hat{X}_p^{-1} \hat{X}_e^{-1}. \tag{3.1}$$

The metric tensor of the elastic deformations is determined by the relation

$$\hat{g} = (\hat{X}_e^{-1})^* (\hat{X}_e^{-1}).$$

Postulating plastic incompressibility of the material, for the density of the material we have

$$\rho = \rho_0 / \sqrt{\det \hat{g}}.$$

As a result of the equation for $\hat{X}_e$, (3.1) is the equation for the metric tensor

$$d\hat{g}/dt = -(\hat{g}\hat{\Omega}^{\cdot}_p)^* - (\hat{g}\hat{\Omega}^{\cdot}_p).$$

Further, we denote by $\hat{\varepsilon}^{\cdot}_p$ and $\hat{\omega}^{\cdot}_p$ the symmetrical and antisymmetrical parts

$$\hat{\varepsilon}^{\cdot}_p = \frac{1}{2}[\hat{\Omega}^{\cdot}_p + (\hat{\Omega}^{\cdot}_p)^*], \ \hat{\omega}^{\cdot}_p = \frac{1}{2}[\hat{\Omega}^{\cdot}_p - (\hat{\Omega}^{\cdot}_p)^*],$$

where $\hat{\omega}^{\cdot}_p$ is the tensor of the rates of plastic rotations. Using an analysis analogous to that for the function $\hat{J}$ (1.5), it can be shown that, for an isotropic material, $\hat{\omega}^{\cdot}_p = 0$.

In a Euler system of coordinates, we write a closed system of equations of motion for a medium with a defect structure:

the equation of the conservation of mass

$$\rho = \rho_0 / \sqrt{\det \hat{g}}; \tag{3.2}$$

the equation of the momenta $\rho d\mathbf{v}/dt + \operatorname{div}\hat{\sigma} = 0$, where v is the vector of the mass velocity;

the equation for the metric tensor

$$d\hat{g}/dt = -\hat{g}\left[\frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\hat{\sigma}'}{\mu}\right] + \left[\hat{g}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\hat{\sigma}'}{\mu}\right)\right]^*;$$

the equation for the tensor of the stresses

$$\hat{\sigma} = \rho \partial U / \partial \hat{\varepsilon}_e;$$

the equation for the shear viscosity

$$\mu = \hat{\tau} / bnu_s(\tau_m, T, n, \varkappa);$$

the ratio for the tensor of the elastic deformations

$$d\hat{\varepsilon}_e/dt = -d\hat{g}/dt;$$

the Orovan equation

$$d\gamma^p_m/dt = bnu_s(\tau_m, T, n, \varkappa);$$

the equation for the entropy

$$ds/dt = \left[\mathrm{Sp}\hat{\sigma}\cdot\hat{\varepsilon}^{\cdot}_p - \rho\left(\frac{\partial U}{\partial n} f_n + \frac{\partial U}{\partial \varkappa} f_\varkappa\right)\right] \Big/ \rho T;$$

the equation for the temperature

$$T = \partial U / \partial s.$$

If the kinetic equations (1.7), (1.8) are given, as well as the function of the internal energy, then the system (3,2) is found to be closed and can be used to determine the parameters ρ, $\hat{\chi}$, v, $\hat{\dot{\chi}}$, μ, γP_m, $\hat{\varepsilon}_p$, s, T, $\hat{\varepsilon}_e$.

4. Let us consider the one-dimensional motion of the medium and write a system of equations describing this motion. Let the medium move parallel to the axis which we designate by the subscript 1, with the velocity $v = v(x, t)$, where x is an Euler coordinate. We note that such a deformation is a deformation at the principal axes, and, in an isotropic medium, the tensors of the stresses and the elastic deformations are coaxial. Let $(\lambda_1, \lambda_2, \lambda_3)$ and $(\sigma_1, \sigma_2, \sigma_3)$ be, respectively, the elastic elongations and the stresses along the principal axes; here $\lambda_2 = \lambda_3$ and $\sigma_2 = \sigma_3$. For the metric tensor and the tensor of the stresses, we have [8]

$$\hat{g} = \begin{vmatrix} \lambda_1^{-2} & 0 & 0 \\ 0 & \lambda_2^{-2} & 0 \\ 0 & 0 & \lambda_3^{-2} \end{vmatrix}, \quad \hat{\sigma} = \begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}.$$

Introducing, instead of the elongations λ_1, λ_2, λ_3, the components of the tensor of the elastic deformations

$$\varepsilon_{1e} = \ln \lambda_1, \ \varepsilon_{2e} = \ln \lambda_2, \ \varepsilon_{3e} = \ln \lambda_3,$$

we obtain the results that, for the one-dimensional case, the system of equations is written in the following form:

the equation for the density of the material

$$\rho = \rho_0 \exp(-\varepsilon_{1e} - 2\varepsilon_{2e}); \tag{4.1}$$

the equation of the momenta

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) - \frac{\partial \sigma_1}{\partial x} = 0; \tag{4.2}$$

the equation for the component of the tensor of the elastic deformations

$$\frac{\partial \varepsilon_{1e}}{\partial t} + v\frac{\partial \varepsilon_{1e}}{\partial x} = \frac{\partial v}{\partial x} - \frac{\sigma_1'}{\mu}, \ \frac{\partial \varepsilon_{2e}}{\partial t} + v\frac{\partial \varepsilon_{2e}}{\partial x} = -\frac{\sigma_2'}{\mu}; \tag{4.3}$$

the relationship for the shear viscosity

$$\mu = \tau / bnu_s(\tau, T, n, \varkappa); \tag{4.4}$$

the equation for the entropy

$$\rho T\left(\frac{\partial s}{\partial t} + v\frac{\partial s}{\partial x}\right) = \mathrm{Sp}\hat{\sigma}\cdot\hat{\dot{\varepsilon}}_p - \rho\left(\frac{\partial U}{\partial n} f_n\right); \tag{4.5}$$

the equation of state

$$\sigma_1 = \rho\partial U/\partial\varepsilon_{1e}, \ \sigma_2 = \rho\partial U/\partial\varepsilon_{2e}, \ T = \partial U/\partial s. \tag{4.6}$$

Here it is assumed that $\varkappa$ = const. For large deformation rates, which are attained with high-speed deformation of materials, we can use the dependence between the mean rate of the dislocations u_s, the maximal shear stress τ, and the mean density of the dislocations n, proposed in [9]:

$$u_s = c_s \exp\left(-\frac{\tau_*}{\tau} - \frac{n}{n_*}\right), \tag{4.7}$$

where c_s is the elastic transverse speed of sound; τ_* and n_* are kinetic parameters, depending on the previous treatment of the material (annealing, irradiation, etc.), the temperature, and the granularity. In the case of high-speed loading, in the stage of active loading (at the front of the shock wave), diffusional mechanisms of the motion of the dislocations are not able to develop to a sufficient degree and the kinetic equation (1.7) is written in the form

$$dn/dt = m(T, \varkappa) bnu_s(\tau, T, n, \varkappa). \tag{4.8}$$

The dependences of the kinetic parameters on the granularity and the temperature are determined from the analysis of one-dimensional problems, analogously to [1]. If these dependences are known, then, the system (4.1)-(4.8) is found to be closed. In the case T = const and $\varkappa$ = const, in [1, 2] a numerical calculation of the system (4.1)-(4.8) is given, with application to the problem of a plane collision between plates.

LITERATURE CITED

1. R. I. Nigmatulin and N. N. Kholin, "A model of an elasticoplastic medium with dislocation kinetics of plastic deformation," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1974).
2. R. I. Nigmatulin and N. N. Kholin, "The dislocation kinetics of superplasticity and creep," Dokl. Akad. Nauk SSSR, 231, No. 2 (1976).
3. N. S. Kozin, A. A. Tuzovskii, and N. N. Kholin, "Determination of the equation of dynamic plasticity," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1978).
4. T. Mura, Continuum Theory of Dislocations and Plasticity, Springer, Berlin (1967).
5. L. I. Sedov, The Mechanics of Continuous Media [in Russian], Vol. 2, Nauka, Moscow (1970).
6. I. I. Goldenblatt, Questions in the Mechanics of Deformable Media [Russian translation], GITTL, Moscow (1955).
7. S. H. Grandal and N. C. Dahl, An Introduction to the Mechanics of Solids, McGraw-Hill, New York (1959).
8. S. K. Godunov, Elements of the Mechanics of Continuous Media [in Russian], Nauka, Moscow (1978).
9. J. Gilman, "Dislocation mobility in crystals," J. Appl. Phys., 36, No. 10 (1965).